



Numerical and statistical analysis on unsteady magnetohydrodynamic convection in a semi-circular enclosure filled with ferrofluid



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ABSTRACT

In this paper unsteady magnetohydrodynamic convection has been analyzed using numerical and statistical techniques for a semicircular-shaped enclosure filled with ferrofluid. Cobalt–kerosene ferrofluid is considered for the present investigation. Galerkin weighted residuals method of finite element analysis is adopted for the numerical simulation. The effects of Rayleigh number (Ra), solid volume fraction (ϕ) of ferrofluid and Hartmann number (Ha) are considered as pertinent parameters and varied for a wide range of values ($Ra = 10^5$ – 10^7 , $\phi = 0$ – 0.15 , $Ha = 0$ – 50) to capture the flow and thermal interaction phenomena for unsteady situation. It is observed that higher ferrofluid solid volume fraction escalates the heat transfer rate. Enhancing the intensity of external magnetic field (by increasing Ha) retards the heat transfer rate while increment of Ra exhibits improvement on the heat transfer. It is also found that there exists a strong interaction between ferromagnetic particle (cobalt) and base fluid (kerosene) in the presence of magnetic field which can be utilized properly for desired heat transfer augmentation. If $\phi = 0$ (or $\phi = 0.15$) and Ra is changed from 10^5 to 10^7 , the linear dependence of $\Psi_{\max}$ on time changes significantly from positive to negative.

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1. Introduction

Convection heat transfer has been given an enormous importance by different researchers as this is a very common phenomenon for transferring heat. Many numerical simulations and experimental works have been carried out to understand these heat transfer phenomena properly. In some cases of practical applications, there are some deviations in the result due to not considering some important parameters. Magnetic field affects convective heat transfer phenomena significantly. So considering this factor in modeling is important and in recent years different articles are published on effect of magnetic field in convection heat transfer.

Basically when the fluid flows due to convection in presence of any magnetic field, it causes Lorentz force in the fluid which reduces the velocity of the fluid particles. Magneto hydrodynamic (MHD) flow has a vast range of applications such as crystal growth process, hemodialysis material, boiler, energy storage process,

manufacturing technology solar technologies, food and chemical processing and so on. In phase change applications, MHD convection plays a vital role which is implemented largely in materials engineering [1]. Crystal growth can be made better using magnetic flux [2,3].

Different techniques have been adopted by different researchers to solve the problem numerically, analytically and experimentally. Borghi et al. [4] numerically investigated nonlinear electrodynamics in MHD regimes with magnetic Reynolds number numerically. Abbassi and Nassrallah [5] used finite element method to solve MHD problem. Finite difference method was adopted by Sposito and Icefall [6]. Hadid et al. [7] also studied MHD convection using finite difference method. Aldoss et al. [8] and Alchaar et al. [9] used analytic method in this regard. Related topics on this field can be found in the following literatures [10–19].

Nasrin and Alim [20] used control volume finite element method of MHD on forced and natural convection in a vertical channel with a heat generating pipe. They found that both MHD and joule heating effects reduced the heat transfer rate. Grosan et al. [21] investigated the magnetic fields and internal heat generations effect on the convection in a rectangular cavity which was filled with a porous medium. They reported that alignment of the

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Nomenclature

B_0	magnitude of magnetic field
c_p	specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
g	gravitational acceleration (m s^{-2})
H	enclosure height (m)
Ha	Hartmann number
k	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
L	length of the enclosure (m)
Nu	Nusselt number
p	dimensional pressure ($\text{kg m}^{-1} \text{s}^{-2}$)
P	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
S_δ	source term in Eq. (1)
T	fluid temperature (K)
t	dimensional time (s)
u	horizontal velocity component (m s^{-1})
U	dimensionless horizontal velocity component
v	vertical velocity component (m s^{-1})
V	dimensionless vertical velocity component
x	horizontal coordinate (m)
X	dimensionless horizontal coordinate
y	vertical coordinate (m)
Y	dimensionless vertical coordinate

Greek symbols

α	thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
β	thermal expansion coefficient (K^{-1})
δ	dependent non-dimensional variables
Γ_δ	diffusion term in Eq. (1)
ϕ	solid volume fraction
μ	dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
ν	kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
τ	dimensionless time
θ	non-dimensional temperature
ρ	density (kg m^{-3})
σ	electrical conductivity
ψ	streamfunction

Subscripts

av	average
h	heat source
c	cold
f	fluid
ff	ferrofluid
s	ferroparticle
max	maximum
min	minimum
L	local

magnetic field affected Nusselt number and reduced the heat transfer. Rahman et al. [22] carried out a finite element solution on MHD convection in a channel having partially or fully heated cavity and observed that length of heater was insignificant for the higher Hartman number. Authors also reported that higher Rayleigh number increased the heat transfer rate. Rahman et al. [23] also studied the combined effect of joule heating and MHD effect on double diffusive mixed convection in a horizontal channel with an open cavity. They found that heat transfer was dependent on different parameters like Richardson number, Hartmann number, joule heating, buoyancy ratio and Lewis number. Ozotop et al. [24] investigated the effect of two semicircular heaters on the bottom wall on MHD convection and reported that distance between the heaters played an important role while the increasing of Hartman number plays negative role for the heat transfer. Farid et al. [25] investigated on MHD convection in lid-driven cavity having a heated circular hollow cylinder and concluded that heat transfer was dependent on the Hartman number.

Sivasankaran and Ho [26] investigated temperature dependent properties on MHD convection and reported that heat transfer was nonlinear with density inversion function and the direction of magnetic field affected the heat transfer also. Hassanuzzaman et al. [27] studied MHD convection on trapezoidal cavities and found that the increase of Hartman number decreased the heat transfer rate. Volumetrically and laterally heated square cavities were studied for MHD by Sarris et al. [28] and they concluded that the ratio of external and internal Rayleigh numbers can make the system steady to unsteady and vice versa. Damping effect of Hartman number was not found by them. Singh and Lal [29] studied the effect of magnetic field orientation on MHD convection and observed that the flux was maximum when the applied magnetic field was parallel to diagonal. Cavity shape is an important parameter for enhancing heat transfer rate. Recently Rahman et al. [30] studied the halfmoon shaped cavity for different nanofluids and concluded that up to 30% heat transfer can be enhanced due to halfmoon shape of the cavity.

Now a days it is a common trend to use nanofluid to augment the heat transfer rate. Different types of nanofluids are available

commercially and used successfully for the enhancement of heat transfer. Among them Al_2O_3 –water, TiO_2 –water, Cu–water are very common. Beside these nanofluids, there are some emerging nanofluids like CNT–water and diamond–water which have significant heat transfer enhancement rate. However, these types of nanofluids still do not serve the economic feasibility comparing with the common nanofluids. Of particular interest to the researchers now a days is ferrofluid where ferromagnetic particle is used as a nanoparticle. Fe_3O_4 , cobalt and nickel are extensively used for this purpose and this type of nanofluid showed promising characteristics to enhance the heat transfer rate [31,32]. Mohsen and Ganji [31] studied MHD effect on ferrofluid in a semicircular annulus and concluded that the increment of solid volume fraction of ferrofluid along with Rayleigh number increments played positive role for the augmentation of heat transfer while decreasing Hartman number caused better heat transfer. Jafari et al. [33] investigated cobalt kerosene ferrofluid and reported that ferrofluid has significant effect on magnetohydrodynamic convection. Similar studies on ferrofluid can be found in literatures [34–39] which justifies the use of ferrofluid in MHD convection. For numerical modeling of nanofluid as well as ferrofluid, different models are available. Single phase model, two phase models (Eulerian, VOF etc.) are popular among them. Some researchers have considered effect of agglomeration too. However, for simplification and finding the general trend as prediction to experimental work, single phase model is followed. This model is a very popular one and is used by many famous researchers [40–44].

From the above literature survey, it is evident the heat transfer in a MHD convection for an enclosure filled with ferrofluid depends on different parameters like Hartmann number, Rayleigh number, cavity inclination, solid volume fraction of ferrofluid and so on. Increase of Hartman number shows damping effect in heat transfer and the maximum magnetic effect can be obtained when the magnetic field is aligned to 45° .

The objective of this paper is to find out the effect of unsteady MHD convection in a semicircular enclosure having a bottom heater. A magnetic field is implemented inside the cavity and the effect of the magnetic flux has been studied. As this is a very

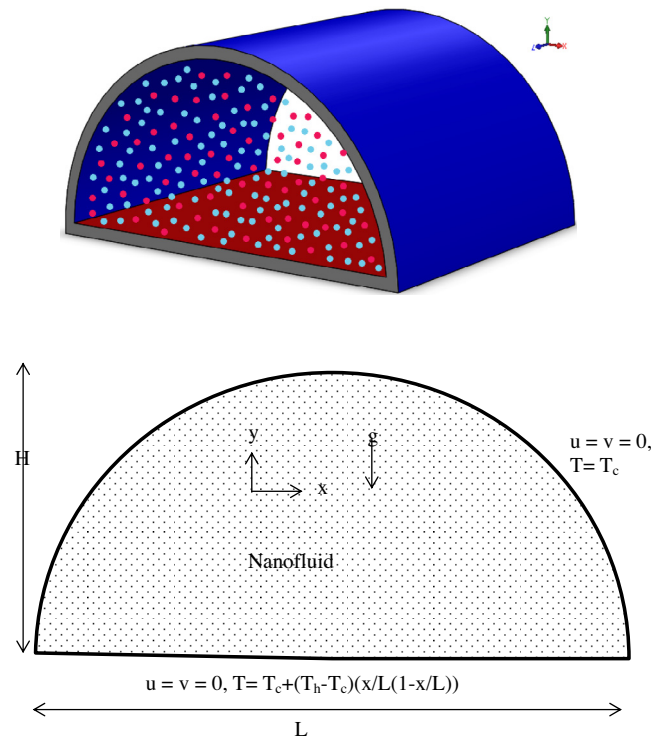


Fig. 1. 3D view (top) and 2D view (bottom) of the schematic of the problem with the domain and boundary conditions.

practical phenomenon in solar thermal collector, this work has special importance. This paper encompasses the effect of different parameters such as Hartman number, solid volume fraction of the ferrofluid and Rayleigh number. The results of the study are presented with the help of isotherm contours, streamlines, Nusselt number, stream function value varying with dimensionless time. A statistical analysis has been done to investigate the linear dependence of average Nusselt number and maximum value stream function on time for varying ϕ and Ra .

2. Problem formulation

2.1. Physical modeling

The details of the physical problem are shown in the Fig. 1 with the well defined coordinate system along with the specified boundary conditions. Basically the enclosure is a semicircular shaped cavity having bottom length L . The height of the cavity is

Table 3
Result of grid independence test.

Number of grid elements	Nu_{av}
1242	0.994875
2620	1.024563
3206	1.104692
4628	1.197233
5420	1.198547
6251	1.201451

H ($H = L/2$). The bottom of the enclosure is heated non-uniformly and the round wall is kept at low temperature. The entire cavity is filled with cobalt–kerosene ferrofluid. An external magnetic field (B_0) is applied to the x -direction of the enclosure. The effect of the gravity is specified in the negative direction of y -axis. Radiation mode of heat transfer is not considered for the physical problem.

This is a very common physical problem in different engineering applications. It may be very analogical with the heat transfer in the solar collector where the semicircular annular wall is the glass cover and the bottom plate is the heated plate for the collector. However, this physical problem has relevance with other applications where magnetic field is applied or created due to the flow of the fluid.

2.2. Mathematical modeling

The two-dimensional steady state continuity, momentum and energy equations are applied to model the problem for flow and thermal fields. The working fluid is assumed to be incompressible, Newtonian with constant properties and thermal equilibrium between the ferromagnetic particles and the base fluid are assumed. Boussinesq approximation is applied for the density variation of the fluid. From the above stated assumptions, the non-dimensional governing equations take the form as [30],

$$\frac{\partial(\delta)}{\partial\tau} + \frac{\partial(U\delta)}{\partial X} + \frac{\partial(V\delta)}{\partial Y} = \frac{\partial}{\partial X} \left(\Gamma_\delta \frac{\partial\delta}{\partial X} \right) + \frac{\partial}{\partial Y} \left(\Gamma_\delta \frac{\partial\delta}{\partial Y} \right) + S_\delta. \tag{1}$$

Here non-dimensional dependent variables are designated by δ and corresponding diffusion and source terms are defined by Γ_δ and S_δ , respectively and those are summarized in Table 1.

Scales which are adopted to obtain the above non-dimensional governing equation are presented below

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \tau = \frac{\alpha_f t}{L^2}, \quad U = \frac{uL}{\alpha_f}, \quad V = \frac{vL}{\alpha_f},$$
$$P = \frac{(p + \rho_f g y)L^2}{\rho_f \alpha_f^2}, \quad \Theta = \frac{(T - T_c)}{(T_h - T_c)}. \tag{2}$$

Table 1
A summary of the terms of the non-dimensional governing equations (1) [30].

Equations	δ	Γ_δ	S_δ
Continuity	1	0	0
U -momentum	U	$\mu_{nf}/\rho_{nf}\alpha_f$	$-\partial P/\partial X$
V -momentum	V	$\mu_{nf}/\rho_{nf}\alpha_f$	$-\partial P/\partial Y + (\rho\beta)_{nf}/(\beta_f\rho_{nf})RaPr\Theta - (\sigma_{nf}\rho_f)/(\rho_{nf}\sigma_f)Ha^2PrV$
Thermal energy	Θ	α_{nf}/α_f	0

Table 2
Thermophysical properties of kerosene and cobalt [38,48].

	c_p (J kg ⁻¹ K ⁻¹)	ρ (kg m ⁻³)	k (W m ⁻¹ K ⁻¹)	β (K ⁻¹)	σ (Simens/m)
Fluid: kerosene	2090	780	0.149	9.9×10^{-4}	6×10^{-10}
Solid: cobalt	420	8900	100	1.3×10^{-5}	1.602×10^7

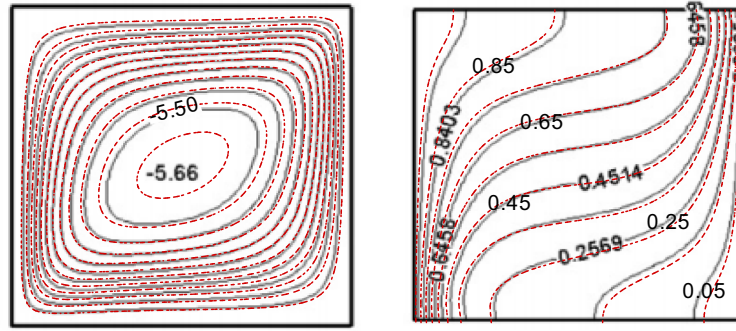


Fig. 2. Comparison of streamline, isotherm for $\phi = 0.03$, $Ra = 10^5$ and $Ha = 30$ with Ghasemi et al. [47]. Red dotted line indicates the present code. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

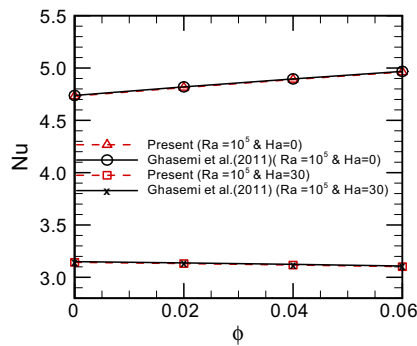


Fig. 3. Comparison of average Nusselt number as a function of solid volume fraction for $Ha = 0$ and 30 at $Ra = 10^5$ with Ghasemi et al. [47]. Red dotted line indicates the present code. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

In the aforementioned equations, U , V , P and Θ are non-dimensional velocities, pressure and temperature respectively. Here subscripts 'ff' and 'f' stand for the properties of the ferrofluid and the base fluid respectively.

The non-dimensional governing parameters Rayleigh number (Ra), Prandtl number (Pr) and Hartmann number (Ha) can be defined as

$$Ra = \frac{g\beta_f L^3 (T_h - T_c)}{\alpha_f \nu_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad Ha = B_o L \sqrt{\frac{\sigma_f}{\mu_f}}, \quad (3)$$

where σ_f indicates the electrical conductivity of the fluid.

The density of ferrofluid which is assumed to be constant can be expressed as,

$$\rho_{ff} = (1 - \phi)\rho_f + \phi\rho_s. \quad (4)$$

Here the property of solid ferromagnetic particles (cobalt) is represented by the subscript 's'. In the above equation, solid volume fraction (ϕ) has significant effect on the thermal diffusivity of ferrofluid which is quite different from the base fluid and can be modeled as,

$$\alpha_{ff} = \frac{k_{ff}}{(\rho c_p)_{ff}}, \quad (5)$$

where heat capacitance of ferrofluid $(\rho c_p)_{ff}$ can be found by,

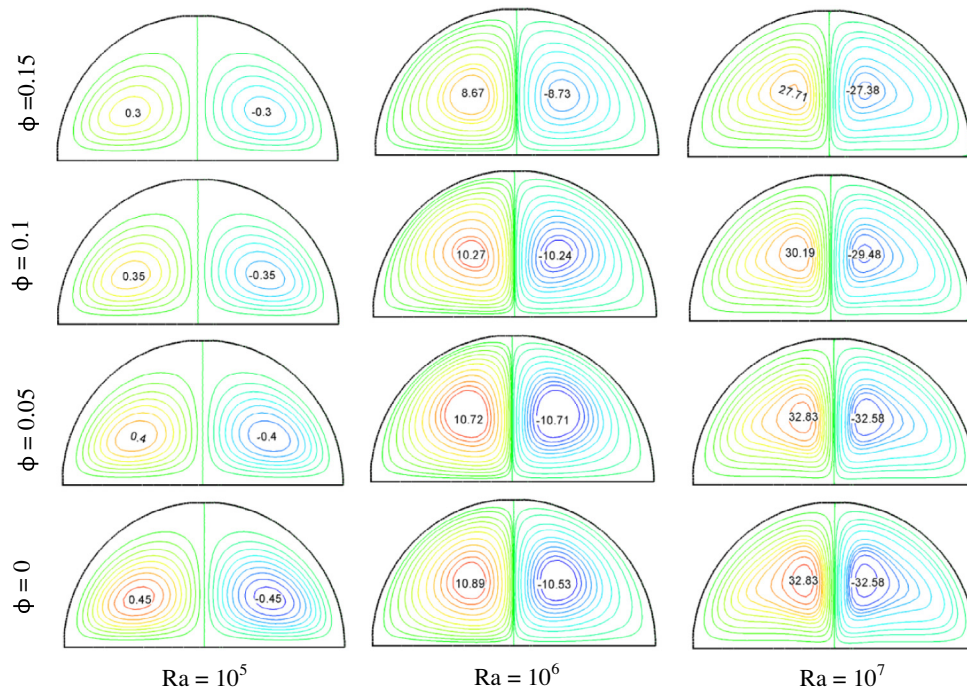


Fig. 4. Effect of solid volume fraction on streamlines for the selected values of Ra and fixed $Ha = 10$ with $\tau = 0.1$.

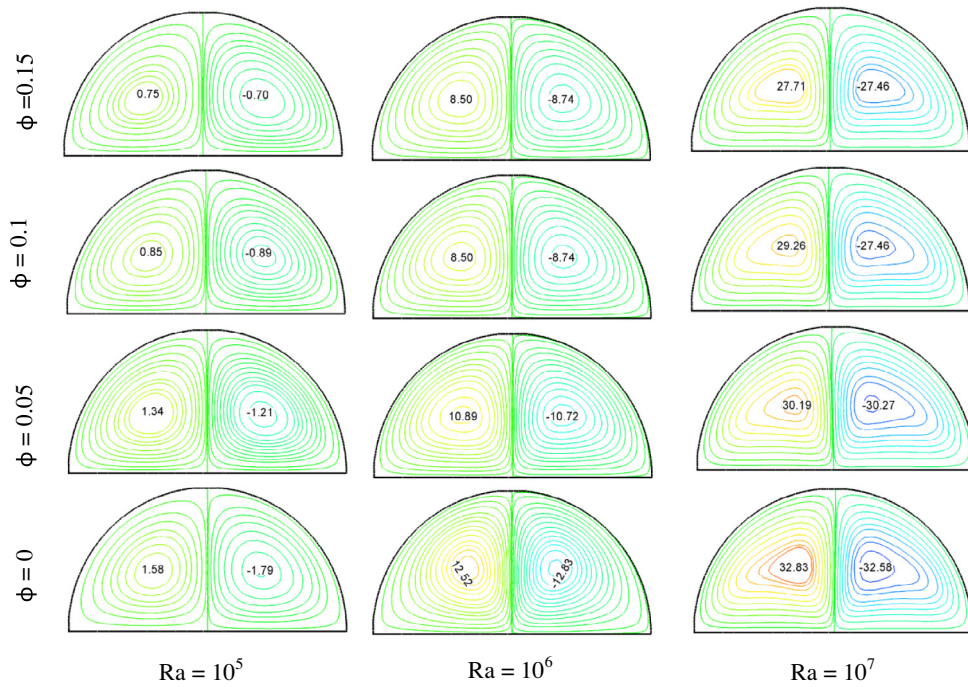


Fig. 5. Effect of solid volume fraction on streamlines for the selected values of Ra and fixed $Ha = 10$ with $\tau = 1$.

$$(\rho c_p)_{ff} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s. \quad (6)$$

In addition, the thermal expansion coefficient (β_{ff}) of the ferrofluid can be obtained as

$$(\rho \beta)_{ff} = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_s. \quad (7)$$

Moreover, dynamic viscosity of the ferrofluid (μ_{ff}) can be expressed using the Brinkman model [45] as,

$$\mu_{ff} = \frac{\mu_f}{(1 - \phi)^{2.5}}. \quad (8)$$

The effective thermal conductivity of the ferrofluid can be described using the Maxwell model [46] as,

$$\frac{k_{ff}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}, \quad (9)$$

and the effective electrical conductivity of the ferrofluid can be described using the Maxwell model [47] as

$$\frac{\sigma_{ff}}{\sigma_f} = 1 + \frac{3(\sigma_s/\sigma_f - 1)\phi}{(\sigma_s/\sigma_f + 2) - (\sigma_s/\sigma_f - 1)\phi}. \quad (10)$$

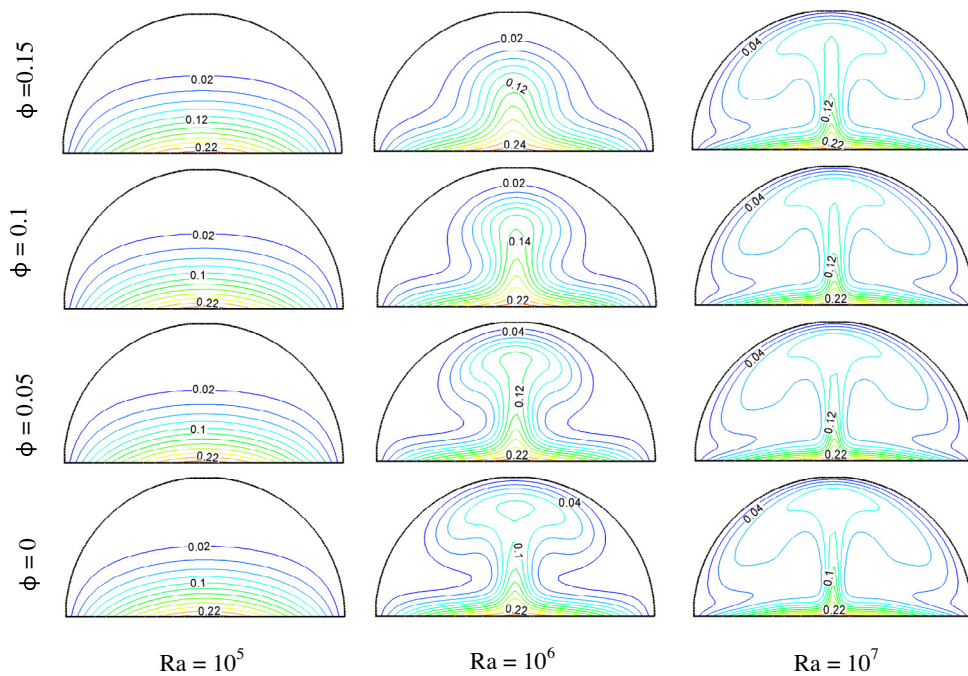


Fig. 6. Effect of solid volume fraction on isotherms for the selected values of Ra and fixed $Ha = 10$ with $\tau = 0.1$.

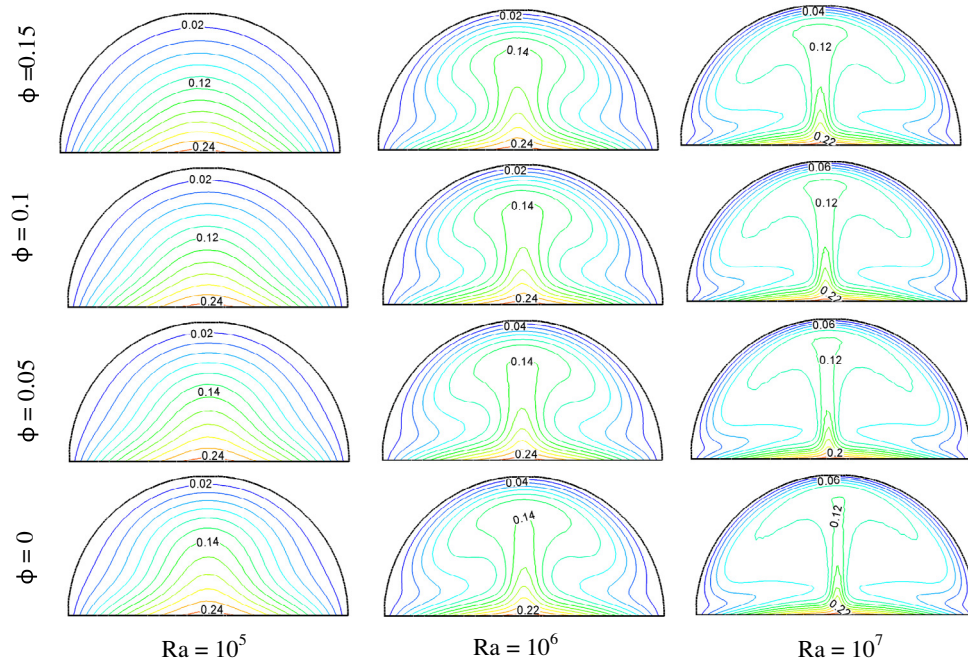


Fig. 7. Effect of solid volume fraction on isotherms for the selected values of Ra and fixed $Ha = 10$ with $\tau = 1$.

Initial and boundary conditions in dimensionless form for the present problems can be defined by

$$\tau = 0,$$

$$\text{Entire domain: } U = V = 0, \Theta = 0$$

$$\tau > 1,$$

$$\text{On the horizontal wall: } U = V = 0, \Theta = 1 + X(1 - X) \quad (11a)$$

$$\text{On the round walls: } U = V = 0, \Theta = 0 \quad (11b)$$

The local Nusselt number on the heat source surface can be expressed as

$$Nu_L = -\frac{k_{ff}}{k_f} \frac{\partial \Theta}{\partial Y}.$$

The average Nusselt number is evaluated by integrating Nu_L along the heat source

$$Nu_{av} = -\frac{k_{ff}}{k_f} \int_0^1 \frac{\partial \Theta}{\partial Y} dX. \quad (12)$$

Flow field of the present problem is visualized through streamline which is obtained from stream function. Stream function is defined from velocity components U and V . Relation between the stream function and velocity components for a two-dimensional flow is given by,

$$U = \frac{\partial \Psi}{\partial Y}, \quad V = -\frac{\partial \Psi}{\partial X}. \quad (13)$$

2.3. Ferrofluids

Ferrofluids are such nanofluids where the ferromagnetic particles such as magnetite, cobalt, nickle etc., having a very good magnetic susceptibility are used as nanoparticle. Base fluids are conventional fluids like water, air, glycerene, kerosene and so on. For the present study, cobalt kersone ferrofluid is used which is a very popular ferrofluid and shows better heat transfer characteristic compared to other available nanofluids. Beside this, cobalt kerosene ferrofluid is less costly and environmental friendly. The

thermophysical properties of the base fluid (kerosene) and the ferromagnetic particle (cobalt) [48] are given in Table 2.

3. Computational details

3.1. Numerical procedure

The Galerkin weighted residuals method of FEM has been used for the numerical scheme of the specified problem. The coupled governing equations were converted into sets of algebraic equations by means of discretizing the entire domain into triangular mesh elements. The second order triangular type elements were used for solving Navier Stokes equation while first order triangular elements are used to solve energy equation. The dependent variables are estimated over each element using quadratic and linear shape functions for flow and thermal fields respectively. Element property equations are converted to global matrix equation and boundary conditions are applied thereafter. Iterative technique is used to solve the resulting algebraic equations. The solution process was sustained until the required convergent criterion was fulfilled which was $|\delta^{m+1} - \delta^m| \leq 10^{-5}$, where m and δ symbolize the number of iteration and the general dependent variable, respectively.

3.2. Grid independency test

To allow grid independent test, the numerical procedure has been conducted for different grid resolutions. Table 3 demonstrates the influence of number of grid elements for a test case of ferrofluid confined within the present configuration. The results show that the grid system of 4628 elements is good enough to obtain accurate results.

3.3. Code validation

A code validation is performed for checking the reliability of the present code. The present code is compared with the results of Ghasemi et al. [47]. The streamline and isotherms are compared

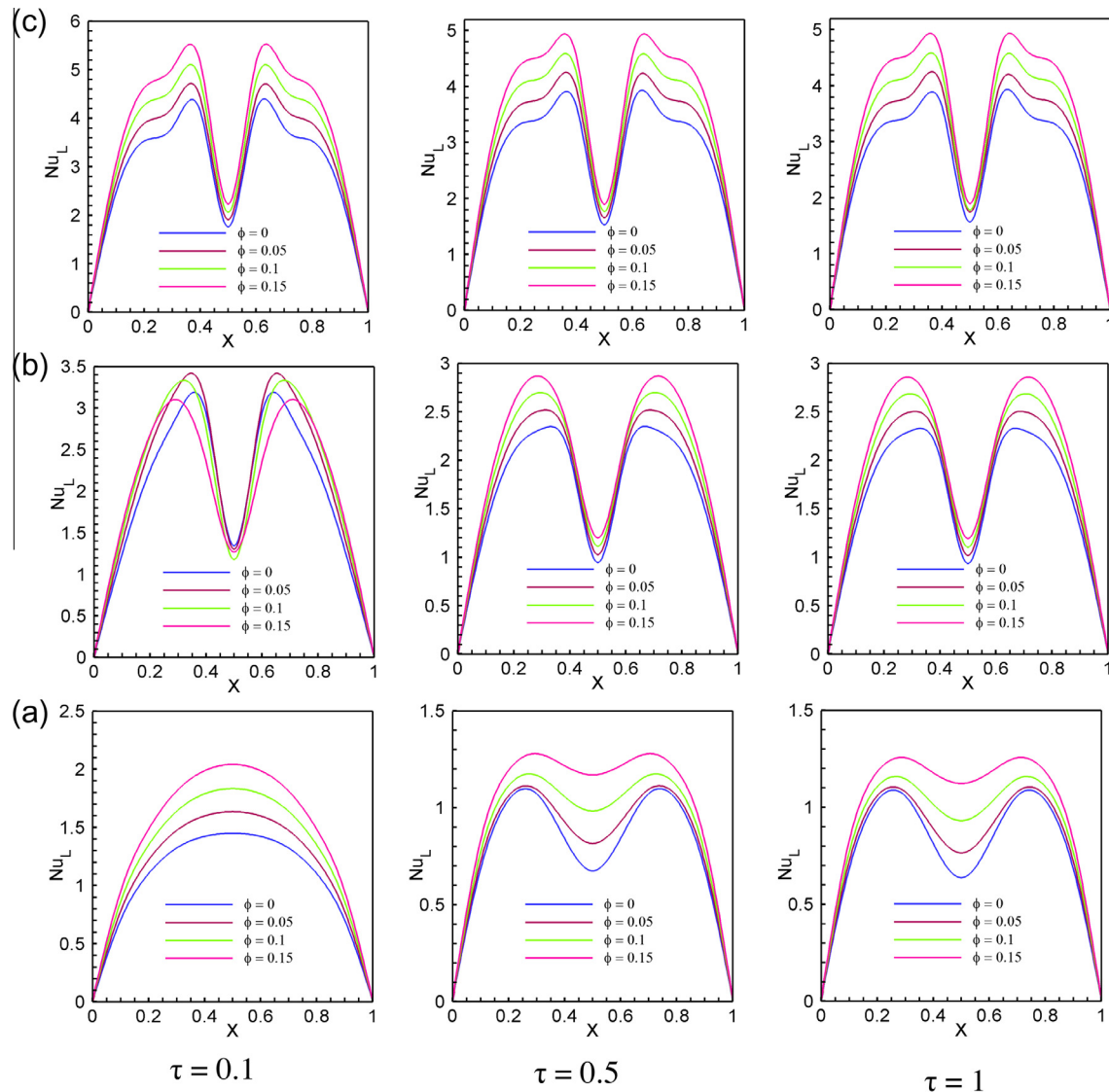


Fig. 8. Effect of the nanoparticle volume fraction and dimensionless time on local Nusselt number at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for fixed $Ha = 10$.

for $Ra = 10^5$, $Ha = 30$ at $\phi = 0.03$ as shown in Fig. 2. From the figure it is evident that the streamline and isotherm produced by the present code is matched with previously published work by Ghasemi et al. [47]. Also the Nusselt number of that literature is compared for two different cases as shown in Fig. 3. The error is very negligible if the Nusselt number is compared. From this point of view it can be said that the present numerical code is completely reliable.

4. Result and discussion

Present analysis tries to introduce some important insights on convective heat transfer in ferrofluid contained cavity under severe convective condition and external magnetic field. To illustrate the evolution of heat transfer, the analysis is made transient and representative cases are presented to discuss the effects of changing Rayleigh, Hartmann numbers and solid volume fraction of ferrofluid upon changing streamlines, isotherms pattern along with local and average Nusselt numbers and stream function value.

Figs. 4 and 5 depict the influence of adding ferrofluid to the fluid inside the cavity by showing streamline maps against Ra (10^5 , 10^6 , 10^7), τ (0.1, 1) and $Ha = 10$. Here, two values of

non-dimensional time have been considered. For $\tau = 0.1$, for every case of Ra , it is seen that there are two oppositely rotating vortices inside the cavity. This pattern is the result of thermal boundary condition at the bottom wall. Fluid inside the cavity after getting heated by the bottom wall tries to go upwards due to influence of buoyant force while relatively cooler fluid near cold sidewalls approaches to bottom wall. Thus a symmetric flow pattern is created. This pattern remains the same irrespective of Ra . However, strength of fluid currents increases with Ra as higher Ra results in higher convective force. Another interesting observation is, with the addition of ferrofluids, streamfunction value is decreasing at any particular Ra . This phenomenon can be described by the fact that the addition of ferrofluids increases the total mass of the fluid inside the cavity which is reflected as higher inertia of the fluid. This higher inertia is causing the fluid to slow down a bit. Now the flow pattern remains the same for both $\tau = 0.1$ and $\tau = 1$. This means, the fluid inside cavity stabilizes very quickly and reaches steady state. This result is useful if the fluid is to be used as fluid sensor for heat detecting unit as the transient response time of heat dispersion inside the fluid is very low.

Figs. 6 and 7 show the isotherm pattern for the present problem under different values of Ra (10^5 , 10^6 , 10^7) and solid volume

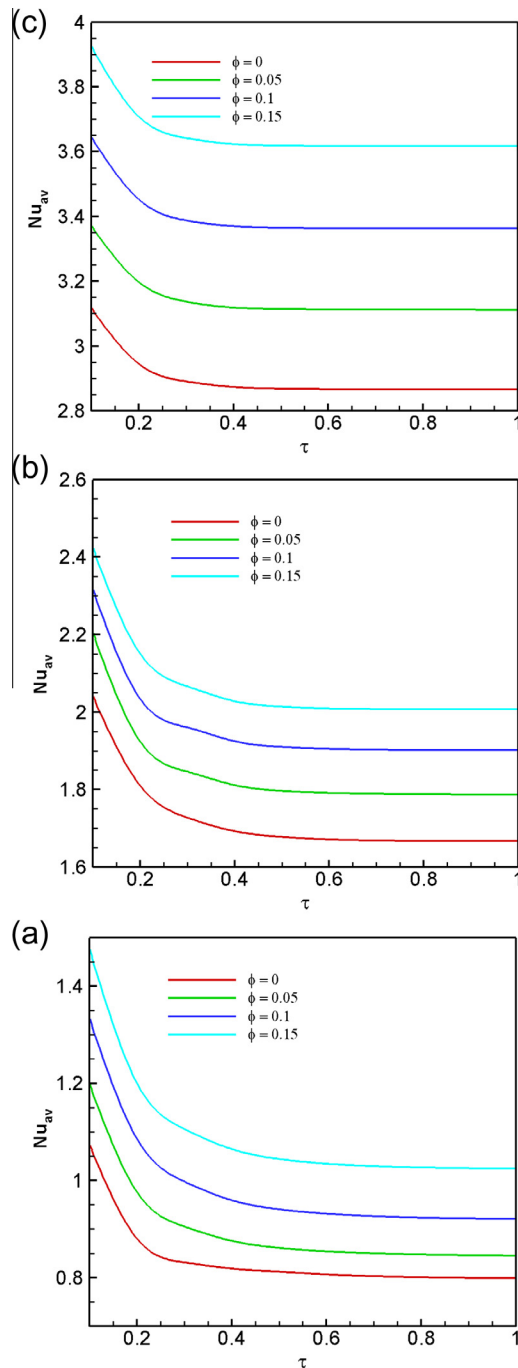


Fig. 9. Effect of the nanoparticle volume fraction and dimensionless time on average Nusselt number at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for fixed $Ha = 10$.

fraction of ferrofluid for two values of τ (0.1 and 1) and $Ha = 10$. Now the isotherms pattern seems to indicate that at low value of Ra , convection is weaker inside the cavity as the isotherms are seen almost parallel to each other near the heated cavity and forming cavity-like arc near the top. This situation does not change with the addition of ferroparticles inside the cavity. But, as the value of Ra increases, the isotherms become more and more distorted near the middle forming a pattern like mushroom. This particular pattern suggests that heat energy is flowing into the fluid inside cavity from bottom heated wall, while, increasing Ra intensifies the effect of Ra number increment. Adding ferroparticle inside the fluid has a very little effect on heat diffusion. However, unlike

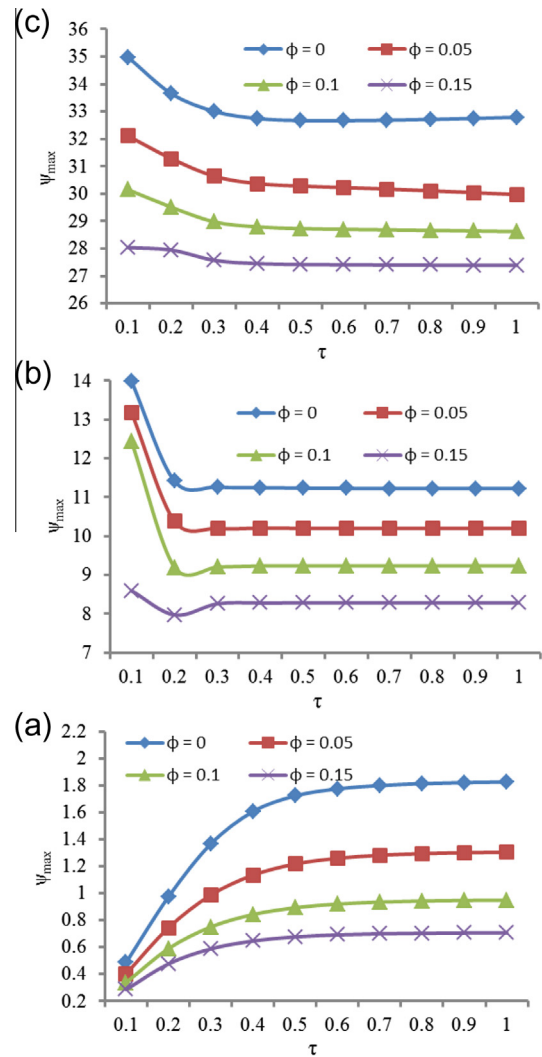


Fig. 10. Effect of the nanoparticle volume fraction and dimensionless time on maximum streamfunction in the cavity at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for fixed $Ha = 10$.

fluid dispersion, heat dispersion requires some time to set in and be diffused in the cavity. At $\tau = 1$, the isotherms are seen more dispersed inside the cavity compared to the $\tau = 0.1$ case. This particular pattern suggests that heat diffusion is slower than matter inside ferrofluid.

Fig. 8 shows variation of local Nusselt numbers along the heated wall for various solid volume fractions and Rayleigh numbers at different non dimensional time parameter. It is observed from the figure that, the pattern of change in local Nusselt number is almost the same for any certain value of τ . This indicates that heat transfer is established in the same way for different values of Rayleigh number and variation is only due to higher temperature gradient resulting from higher Rayleigh number. For $\tau = 0.1$, heat is still propagating and thus the Nusselt number is still high near the source. As τ becomes 0.5 or more, heat transfer is properly established and as a result local Nusselt number shows peaks at two sides of the principal heat source. Two large vortices inside the cavity actually generate this pattern of heat transfer. On the other hand, as the value of Rayleigh number is increased, the value of Nusselt number increases. With the higher value of solid volume fraction, local Nusselt number increases. This result is indicative of positive impact of nanofluid on heat transfer inside the cavity.

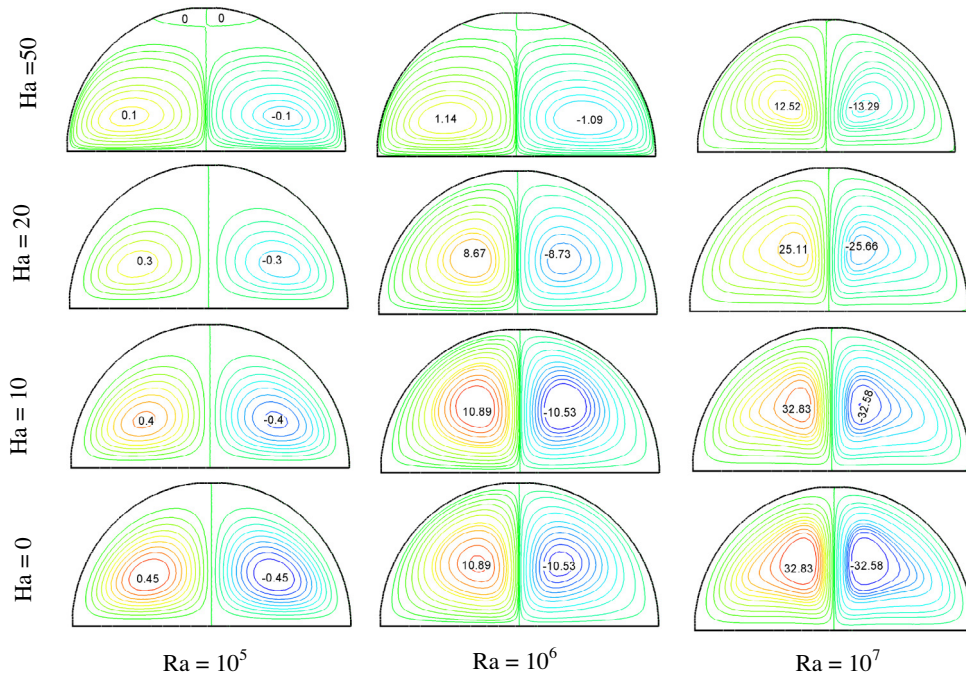


Fig. 11. Effect of Hartmann number on streamlines for the selected values of Ra and fixed $\phi = 0.04$ with $\tau = 0.1$.

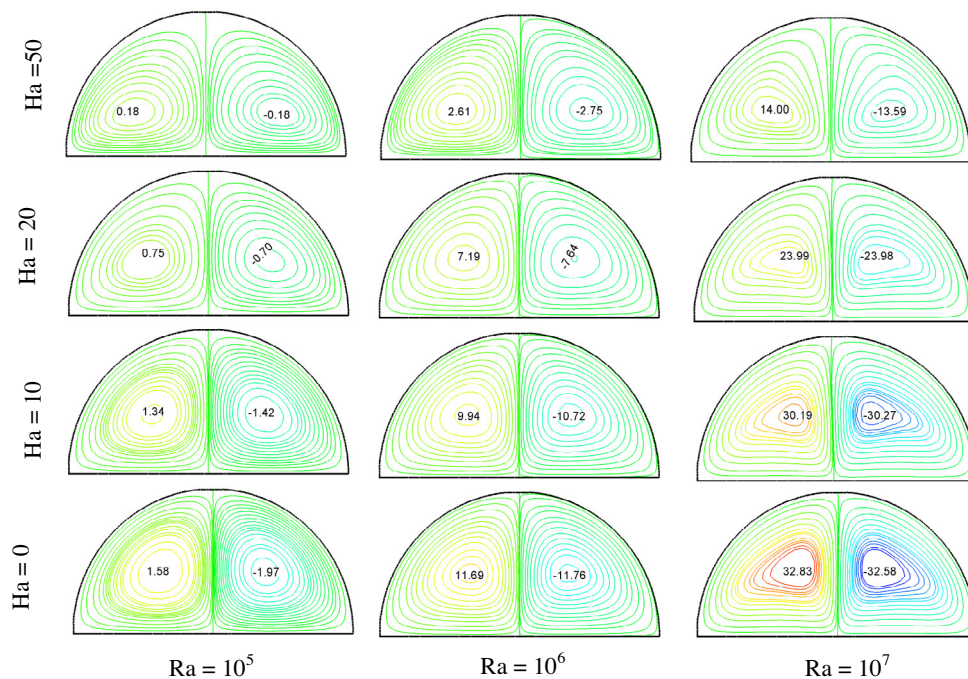


Fig. 12. Effect of Hartmann number on streamlines for the selected values of Ra and fixed $\phi = 0.04$ with $\tau = 1$.

Fig. 9 shows impact of solid volume fraction and Rayleigh number on average Nusselt number with different values of non-dimensional time parameter. As seen previously, Rayleigh number has a positive effect on the value of average heat transfer rate and addition of nanoparticle to the base fluid significantly improves heat transfer. At the beginning of the process, when the flow is unstable, it is seen that, the value of average Nusselt number is quite high and it comes to a more or less constant after reaching steady state.

Fig. 10 exhibits the effect of the nanoparticle volume fraction and dimensionless time on maximum stream function inside the cavity at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for different values of τ and $Ha = 10$. Here some interesting points are observed. For low value of Rayleigh number, strength of vortex increases and then reaches a steady state with time. However, for higher value of Ra , opposite trend is observed. Actually at low Ra values, heat transfer is dominated by conduction process mainly, hence vortex strength is both limited and gradually uprising. In contrast,

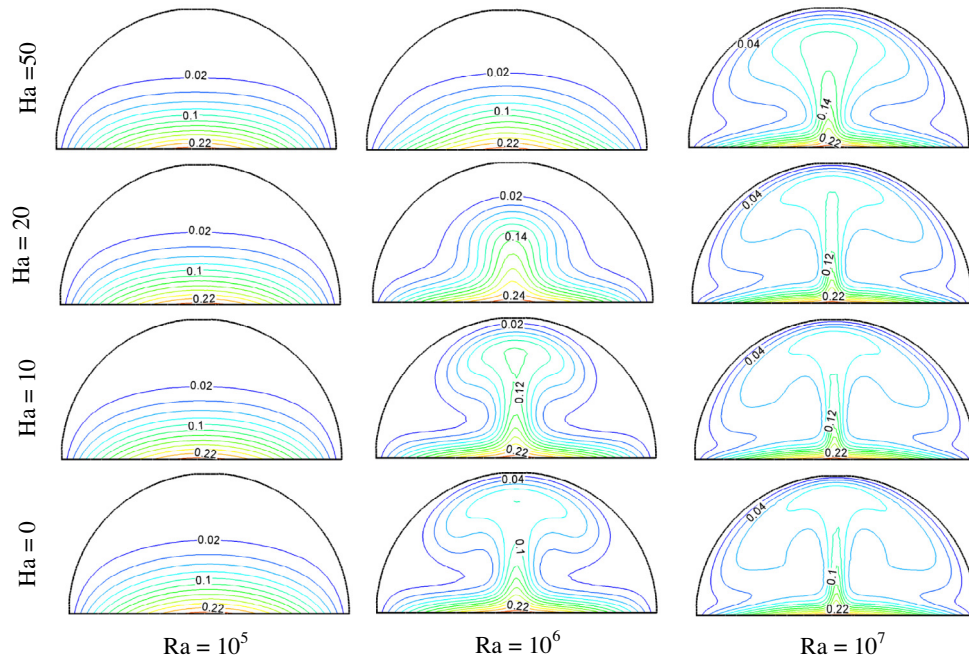


Fig. 13. Effect of Hartmann number on isotherms for the selected values of Ra and fixed $\phi = 0.04$ with $\tau = 0.1$.

at higher thermal gradient, the transport process is dominated by convection rather than conduction. Another interesting outcome is the observation of lower strength of convection currents at higher solid volume fraction. This result can be attributed to the fact that, at higher solid volume fraction, the mass of the flowing stream is also higher. As a consequence, inertia of the flowing fluid compels it to be weak.

Fig. 11 depicts the effect of Hartmann number on streamlines for different values of Ra at $\tau = 0.1$ and $\phi = 0.04$. Actually this figure shows the evolution of streamlines under strong magnetic field and thermal gradient simultaneously. The streamline pattern in

general is similar to what we have already observed. Same basic physical mechanism works here to give rise to two vortices inside the cavity. As the value of Hartmann number is increased (stronger magnetic field is applied), strength of flow inside the cavity decreases. This outcome is supported by many previous researches. Physical reasoning behind this result is that, actually externally applied magnetic field imposes a strong field over moving fluid that has magnetic susceptibility. It results in generating a Lorentz force field which has a nature to oppose the very reason of its generation which in this case is the movement of fluid. This force field weakens the streams inside cavity making the stream

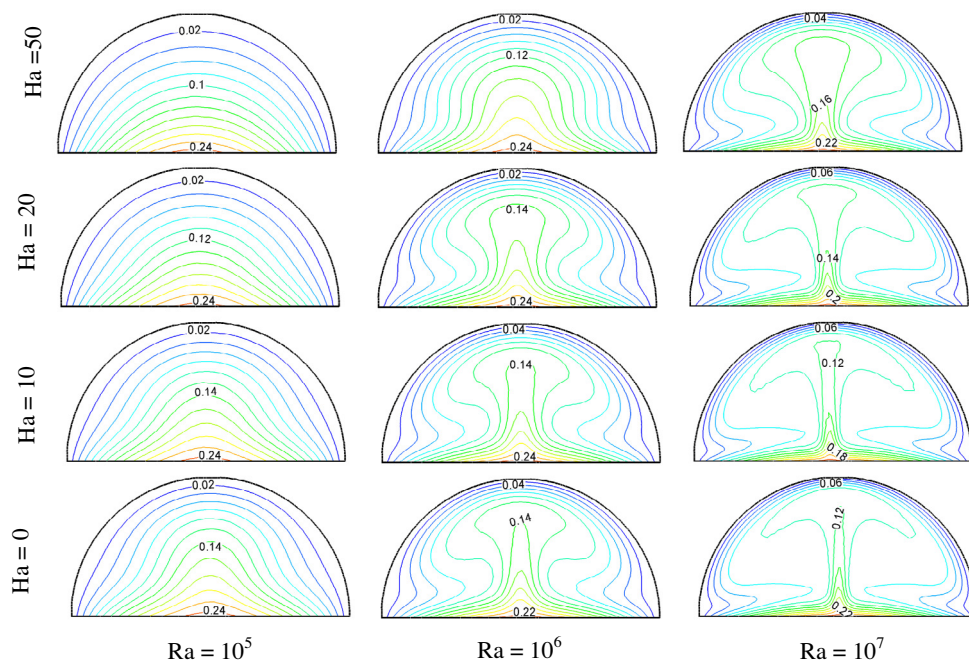


Fig. 14. Effect of Hartmann number on isotherms for the selected values of Ra and fixed $\phi = 0.04$ with $\tau = 1$.

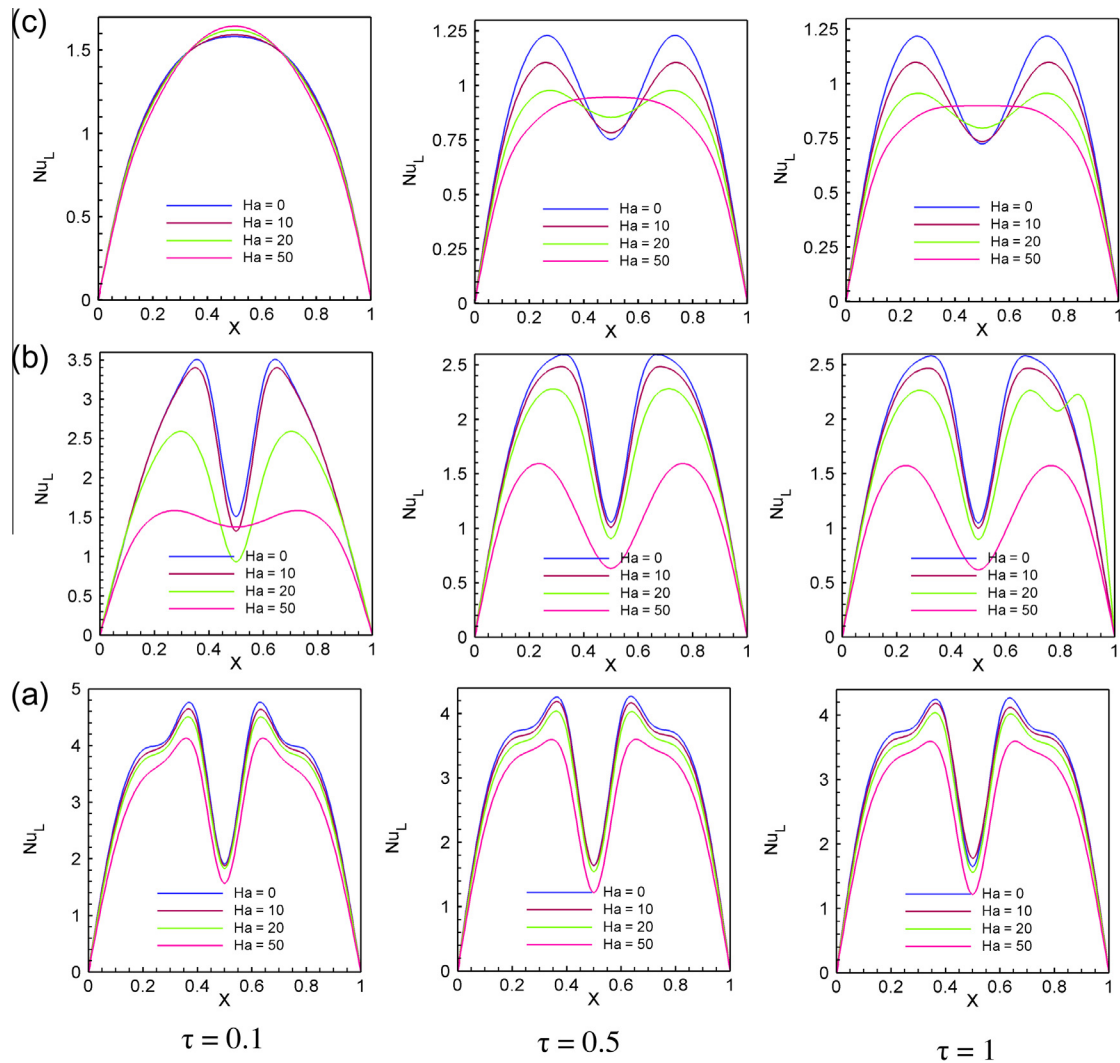


Fig. 15. Effect of the Hartmann number and dimensionless time on local Nusselt number at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for fixed $\phi = 0.04$.

function values down. At very high value of Hartmann number ($Ha = 50$) a portion of fluid near the top wall seems to be completely still. This entire flow pattern is at the onset of the whole process. As we have seen from our previous studies, the process reaches steady state after a finite time. Hence we investigated further to see what happens after the system has reached steady state.

Fig. 12 is the continuation of Fig. 11 that shows the state of streamlines after the system has reached steady state. Influence of Hartmann number and Rayleigh number remain unaltered. Hartmann number is still acting against the flow field and Rayleigh number shows an overall improvement. With time, there is a little increment in the strength of streamlines inside the cavity. Other than that there is not much difference in the flow field.

Figs. 13 and 14 exhibit the effect of Hartmann number on isotherms for different values of Ra at $\tau = 0.1, 1$ and $\phi = 0.04$. From these figures, it can be seen at low values of Ra , isotherms are nearly parallel to the heat source wall. This pattern indicates dominance of conduction near the walls at low values of Ra . As value of Ra increases, isotherms are seen to be disperse and distorted near the middle of cavity. This pattern shows that, convection is beginning to take over and becoming more dominant mode of heat transfer inside the cavity. The same outcome is observed for both $\tau = 0.1$ and 1. Raising Hartmann number is acting against

convection inside the cavity. At high values of Ra , this effect may be not much strong to subside the convection entirely, but even at $Ra = 10^6$, we see that raising Ha to 50 makes the isotherms almost parallel to each other. This clearly indicates that Ha does not only influence the flow field but also retards the thermal field inside the cavity.

Fig. 15 shows the effect of Hartmann number and dimensionless time on local Nusselt number at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ at $\phi = 0.15$. As can be seen from the figure, with increasing Ra , local Nusselt number increases without the influence of Hartmann number. This obviously suggests that, whatever the external field's strength may be, increasing Ra results in better average heat transfer coefficient. Now the ideal distribution of local Nusselt number would be two peaks symmetrically with respect to the center line of the cavity due to the temperature distribution of the heated wall. It is seen that at lower value of Ra , this ideal or steady state distribution is availed at a later stage compared to the other values of Ra . This is due to the fact that, at low Ra , convection takes some time to settle as initial heat transfer is dominated by conduction. Once fluid is heated properly, convection commences. Another important observation is the impact of Ha . Increasing Ha clearly gives lower local Nusselt number in each case. Since stronger external magnetic field weakens the flow field

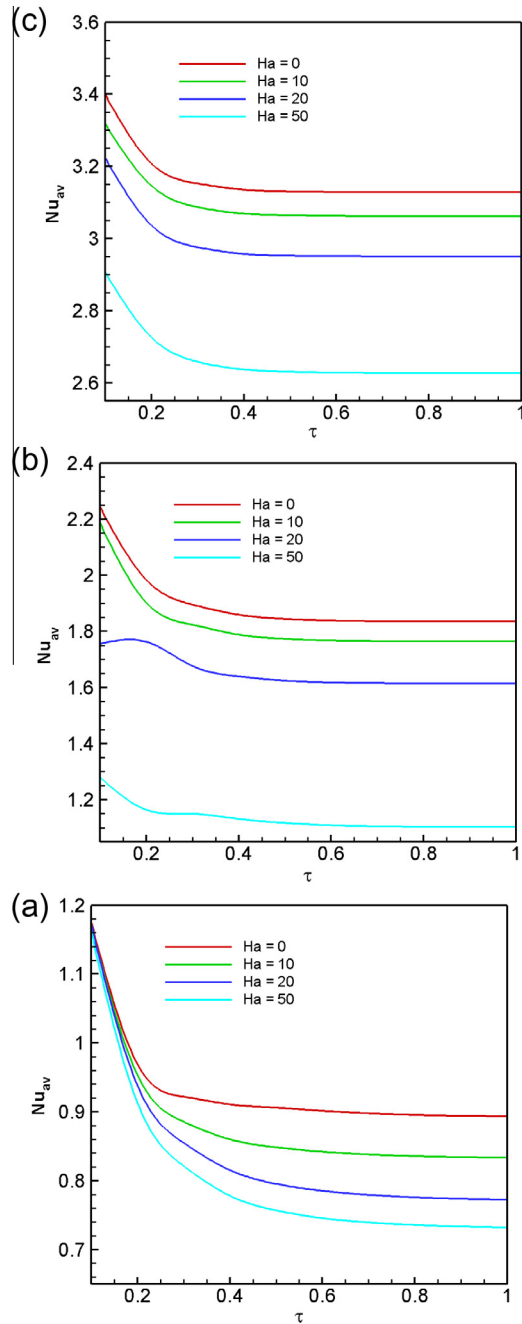


Fig. 16. Effect of the Hartmann number and dimensionless time on average Nusselt number at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for fixed $\phi = 0.04$.

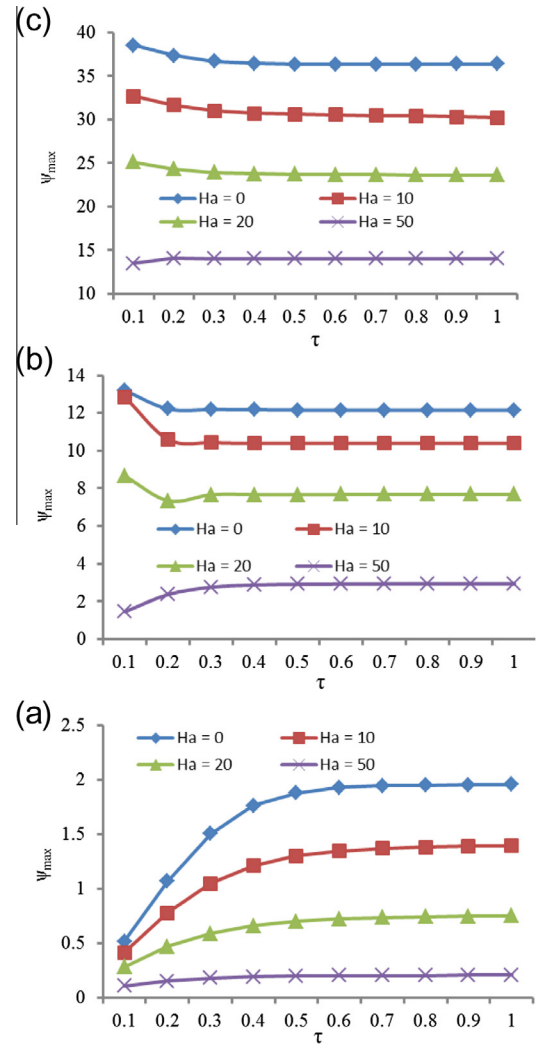


Fig. 17. Effect of the Hartmann number and dimensionless time on maximum streamfunction in the cavity at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for fixed $\phi = 0.04$.

inside the cavity, heat transfer rate decreases with higher value of Ha . This conclusion is supported by many studies mentioned in literature review.

From Fig. 16, we observe the effect of Hartmann number and dimensionless time on average Nusselt number at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for $\phi = 0.04$. As it is seen earlier, raising Ra has produced a positive impact on the average heat transfer rate

Table 4

Analysis of $\psi_{\max}$ with respect to the variation of ϕ and Ra , while fixing $Ha = 10$.

Ra	$\phi = 0$	$\phi = 0.15$	Comments
10^5	Regression: $\psi_{\max} = 0.8237\tau + 1.2636$ Correlation: $r = 0.8436$. Test statistic: $t \approx 4.4436$ p -value ≈ 0.00107879	Regression: $\psi_{\max} = 0.3734\tau + 0.4127$ Correlation: $r = 0.8203$ Test statistic: $t \approx 4.0567$ p -value ≈ 0.00182506	Correlation is significant at 1% level of significance
10^7	Regression: $\psi_{\max} = -1.6783\tau + 33.9877$ Correlation: $r = -0.6903$. Test statistic: $t \approx -2.6986$ p -value ≈ 0.0135664	Regression: $\psi_{\max} = -0.6518\tau + 27.9027$ Correlation: $r = -0.8087$ Test statistic: $t \approx -4.13$ p -value ≈ 0.0013	Correlation is significant at 1% level of significance
Comments	Correlation is significant at 1% level of significance	Correlation is significant at 1% level of significance	

Table 5Analysis of Nu_{av} with respect to the variation of ϕ and Ra , while fixing $Ha = 10$.

Ra	$\phi = 0$	$\phi = 0.15$	Comments
10^5	Regression: $Nu = -0.2084\tau + 0.9634$ Correlation: $r = -0.7412$ Test statistic: $t \approx -3.123$ p -value ≈ 0.007082150	Regression: $Nu = -0.3517\tau + 1.2965$ Correlation: $r = -0.7499$ Test statistic: $t \approx -3.2062$ p -value ≈ 0.0062477	Correlation is significant at 1% level of significance
10^7	Regression: $Nu = -0.1784\tau + 3.0006$ Correlation: $r = -0.6839$ Test statistic: $t \approx -2.6514$ p -value ≈ 0.01459616	Regression: $Nu = -0.2159\tau + 3.7785$ Correlation: $r = -0.6728$ Test statistic: $t \approx -2.5722$ p -value ≈ 0.01650720	Correlation is significant at 2% level of significance
Comments	Correlation is significant at 2% level of significance	Correlation is significant at 2% level of significance	

Table 6Analysis of Nu_{av} with respect to the variation of Ha and Ra while fixing $\phi = 0.04$.

Ra	$Ha = 0$	$Ha = 50$	Comments
10^5	Regression: $Nu = -0.1954\tau + 1.0438$ Correlation: $r = -0.6847$ Test statistic: $t \approx -2.6572$ p -value ≈ 0.01446542	Regression: $Nu = -0.3438\tau + 1.0007$ Correlation: $r = -0.7706$ Test statistic: $t \approx -3.4199$ p -value ≈ 0.00454464	Correlation is significant at 1% level of significance
10^7	Regression: $Nu = -0.189\tau + 3.2707$ Correlation: $r = -0.675$ Test statistic: $t \approx -2.5876$ p -value ≈ 0.01611658	Regression: $Nu = -0.2047\tau + 2.7823$ Correlation: $r = -0.7009$ Test statistic: $t \approx -2.7794$ p -value ≈ 0.01197336	Correlation is significant at 1% level of significance
Comments	Correlation is significant at 1% level of significance	Correlation is significant at 1% level of significance	

inside the cavity. This result is very obvious since raising value of Ra indicates that higher thermal gradient is now applied near the heated wall. Another important observation is that in the transient state Nu_{av} is always higher than steady state value. Increasing Hartmann number has a negative impact on the value of average Nusselt number. Higher Ha results in weaker fluid current inside the cavity and hence lowers heat transfer rate. The physical mechanism behind this is explained earlier.

Fig. 17 shows the effect of varying Hartmann number and dimensionless time on maximum stream function inside the cavity at (a) $Ra = 10^5$, (b) $Ra = 10^6$ and (c) $Ra = 10^7$ for $\phi = 0.15$. Increasing Ra increases the strength of fluid flow inside the cavity. This results in higher value of stream function. From the figure, we can see that higher Ra always results in high value of stream function. Just like we have seen in our study of solid volume fraction and Ra , at low values of Ra , stream function is weak in the transient state and reaches a higher value at steady state. Opposite is seen for high Ra values. Since conduction is dominant heat transfer mode at low values of Ra , it takes time for the fluid flow to establish itself inside the cavity at lower Ra . Raising strength of external magnetic field i.e. Ha , produces weaker streams at each case of Ra . This matches with our previous conclusion that Hartmann number retards the flow inside cavity.

5. Statistical analysis

Analysis of Ψ_{max} with respect to the variation of ϕ and Ra , keeping $Ha = 10$ is shown in Table 4. From the table, it is clear that for $Ra = 10^5$ and $\phi = 0$, the maximum value of Ψ is strongly positively and linearly dependent on time. With $Ra = 10^5$, if ϕ is changed from 0 to 0.15, there is no significant difference in the degree of dependence. Next for $Ra = 10^7$ and $\phi = 0$, the maximum value of Ψ is strongly negatively and linearly dependent on time. With $Ra = 10^7$, if ϕ is changed from 0 to 0.15, there is a perceptible difference in the degree of dependence. On the other hand, at $\phi = 0$, if Ra

is changed from 10^5 to 10^7 , the linear dependence of Ψ_{max} on time changes significantly from positive to negative. The similar trend is observed for $\phi = 0.15$ and if Ra is changed from 10^5 to 10^7 .

Analysis of Nu_{av} with respect to the variation of ϕ and Ra , keeping $Ha = 10$ fixed is shown in Table 5. At $Ra = 10^5$ and $\phi = 0$ (or $\phi = 0.15$), the average Nusselt number is strongly negatively and linearly dependent on time. If ϕ is varied, there is no significant difference in the degree of dependence. Almost similar results are observed at $Ra = 10^7$ for $\phi = 0$ (or, 0.15), except that for smaller Ra , there is a little more negative dependence for lesser value of Ra .

Analysis of Nu_{av} with respect to the variation of Ha and Ra , keeping $\phi = 0.04$ fixed is shown in Table 6. If $Ra = 10^5$ and $Ha = 0$, the average Nusselt number is strongly negatively and linearly dependent on time. The higher the value of Ha , the more is the amount of negative dependence. Similar result is observed for higher $Ra = 10^7$. If $Ha = 0$, the average Nusselt number is strongly negatively and linearly dependent on time for Ra equal 10^5 (or, 10^7).

6. Conclusions

Semicircular shaped enclosures are relatively new in heat transfer studies. Besides, ferrofluids are gaining notice due to their improved physical properties and relatively lower cost. Many heat transfer devices in modern times are run under strong electromagnetic field from some external device. So in this study, the behavior of ferrofluid in a heat transfer scenario under strong external field is investigated thoroughly. Moreover, impact of ferrofluid is studied vividly. Main outcomes of the study can be listed as

- Increasing Ra always aids heat transfer inside cavity. This outcome is independent of the strength of magnetic field. Since in a particular case, temperature difference is quite fixed, this result indicates that we need to modify physical property of a fluid to raise its heat transfer property.

- Higher solid volume fraction of ferroparticle in a base fluid improves its thermo physical properties considerably. The outcome of such modification is higher Nusselt number or better heat transfer rate.
- Higher solid volume fraction slows down the fluid as is seen from Stream function analysis. So there is less wall shear stress developed.
- The heat transfer process for the present problem attains its steady state very fast. We have seen its state unaltered after $\tau = 0.3$.
- Increasing strength of external magnetic field has a negative impact on heat transfer process and relation between Ha and Nu is completely reciprocal. However, high Ha can still produce if Ra is high enough ($Ra = 10^7$). So under strong magnetic field use of ferrofluid is suggested. The study is made considering $\phi = 0.15$ for ferrofluid.
- Stronger magnetic field creates a force field that acts against the fluid flow. This interaction results in lower local and average Nu . Stronger external magnetic field thus has impact on both flow and thermal fields as can be seen from streamline and isotherm plots.
- If $\phi = 0$ (or 0.15) and Ra is changed from 10^5 to 10^7 , the linear dependence of $\Psi_{\max}$ on time changes significantly from positive to negative.

Present study is made with a view to helping the engineering community to develop a better thermo fluid for particular application. Experimental work is recommended for validation of the findings made by the paper.

Conflict of interest

None declared.

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